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Topology of published referral counts among Saudi health clusters: an orientation-invariant audit of the 2024 Ministry of Health matrix

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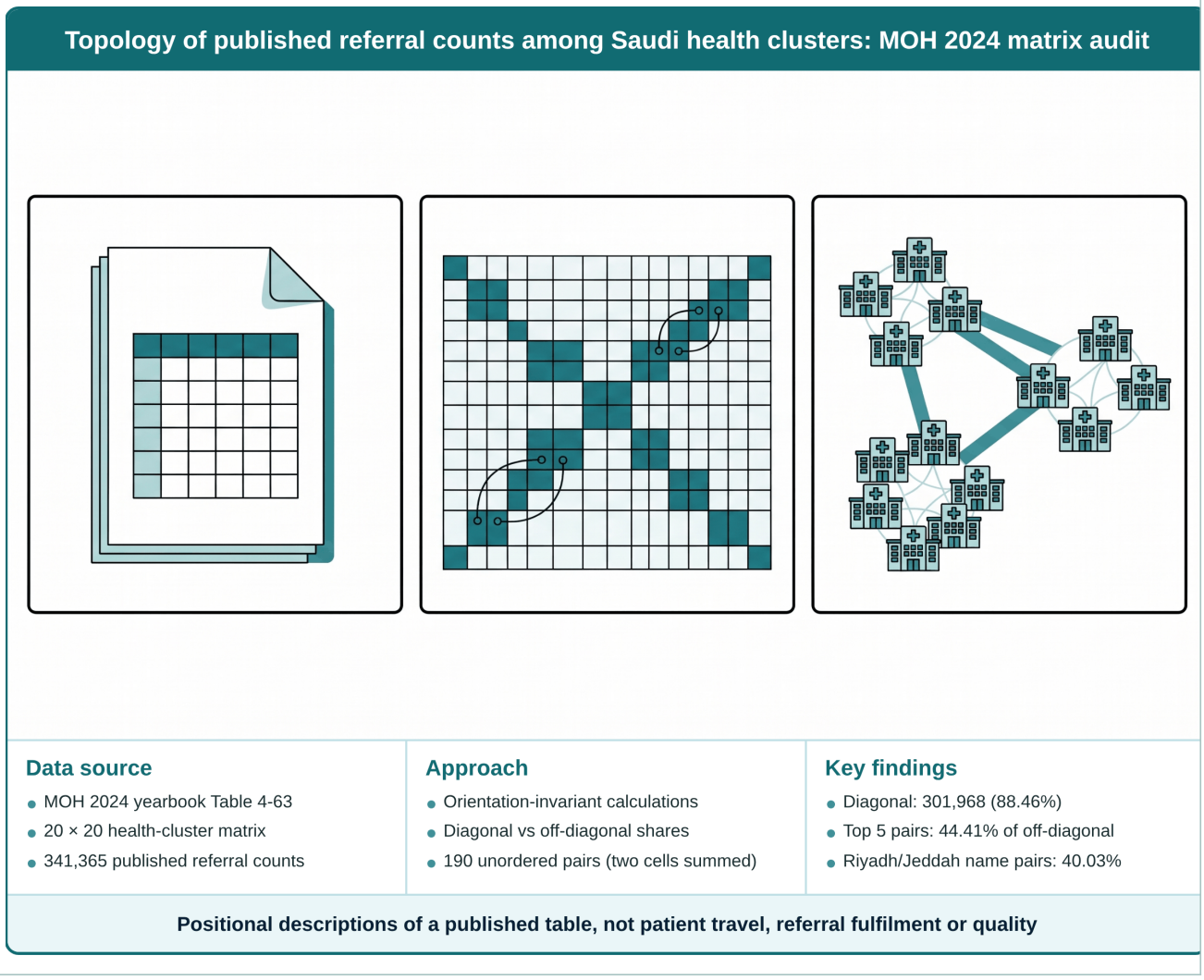
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Abstract

Graphical abstract



Background: The Saudi Ministry of Health (MOH) publishes a cluster-by-cluster table of referrals including reverse referrals. Its two axis headings do not safely establish the direction of a particular cell, and the table does not define whether entries are unique people, requests or completed care. We described its topology using calculations unchanged by transposing the matrix.

Methods: We audited the official 2024 yearbook Table 4-63 as a complete 20 × 20 array. We verified all 400 cells and 40 marginal totals. We calculated diagonal and off-diagonal shares, then combined the two opposing positions for each of 190 unordered pairs of distinct cluster labels. The five largest pairs were an exploratory display cutoff; we also checked ranks one, six and ten and separately identified four pairs whose printed labels share a Riyadh or Jeddah city name. No patient, facility or population denominator was available.

Results: The 400 cells totalled 341,365 published counts. Diagonal cells contributed 301,968 (88.46%); off-diagonal cells contributed 39,397 (11.54%). The five largest unordered pairs contributed 17,496 (44.41% of the off-diagonal total). Fifth-ranked Madinah–Tabouk (1,724) exceeded sixth-ranked Eastern–Al-Ahsa (1,704) by 20; the top six contributed 48.73%. Four pairs between distinct clusters sharing a Riyadh or Jeddah city name contributed 15,772 (40.03% of off-diagonal counts); other off-diagonal counts totalled 23,625 (6.92% of N). These are positional descriptions, not travel or referral-fulfilment measures.

Conclusions: The MOH table permits a reproducible account of concentration among printed cluster labels, especially unordered pairs. It cannot show unique-patient movements, accepted or completed referrals, access, quality or clinical outcomes. Source-owner definitions and orientation are needed before directional or performance analysis.

Keywords: Saudi Arabia; health clusters; referral matrix; reverse referral; administrative data; source provenance; descriptive analysis.

Introduction

The Saudi MOH Statistical Yearbook 2024 publishes a table headed “Referrals including reverse referrals to general or specialized government hospitals according to the referring and receiving cluster in 2024 G” [1]. It has twenty health-cluster labels on each axis, a count at every intersection and printed marginal totals. The apparent geometry invites several very different readings. A diagonal cell may mean that the two printed cluster labels match, but it does not say whether the referring and receiving facilities are the same, whether a person traveled, or whether the referral was accepted. An off-diagonal cell identifies two unlike labels; without a firm axis key it cannot safely be called a transfer from one named cluster to the other. A useful first analysis is therefore one that preserves the table's exact accounting while declining to convert a cell count into a patient or service outcome.

Saudi e-referral research makes the distinction between data frames concrete. Alharbi and colleagues analyzed 755,145 Saudi Medical Appointments and Referrals Center (SMARC) e-referrals from 2023–2024, tabulating 13 administrative regions, business units, internal versus external referral and acceptance-related attributes [2]. Earlier nationwide SMARC analyses described more than 600,000 requests from 2020–2021 [3], modeled referral acceptance in those years [4], and compared referrals within versus between administrative regions [5]. A Saudi primary-care case study investigated an e-referral process and geographic accessibility at its own service level [6]. These studies differ in time frame, event definition, facility scope and geography from the MOH's 2024 twenty-cluster government-hospital table. Their results should not be inserted as missing cells, denominators or validation for it. The 2025 SMARC paper does not present this exact twenty-by-twenty cluster-pair matrix in its accessible full text; that observation defines a narrow analytic opening, not a claim to have invented referral research or discovered that most counts are diagonal.

A 2024 systematic review of e-consultation found possible improvements in access, but low-certainty evidence [7]. This does not establish a mechanism for the MOH table: interpreting concentration as effective gatekeeping would require referral origin, eligibility, disposition and patient follow-up, none of which are available in the MOH array. The more discriminating question is how the off-diagonal published counts are distributed among unordered pairs. This keeps the analysis informative when the row and column direction is uncertain. It also exposes how much of the off-diagonal total is between distinct cluster labels that share a Riyadh or Jeddah city name. Shared wording is not proof of a common catchment, nearby facilities or short travel.

Other network literatures show why data-generating processes matter. A scoping review found that patient-sharing networks assembled from administrative encounters vary in their definitions of a tie and their analytic unit [8]. Analyses of the US patient-referral network [9] and referral paths [10] rely on individual encounter sequences or shared-patient links—evidence not supplied by an aggregate yearbook grid. Hospital postacute referral concentration [11], patient-sharing network costs [12], the persistence of physician relationships [13], and physician- versus patient-induced network construction [14] likewise

address distinct endpoints and data. They motivate precise naming of a matrix edge, not importation of their network statistics or outcome associations into Saudi data.

An actual referral pathway has stages absent from a static count grid. Electronic hospital-referral work has measured information completeness [15], direct-admission coordination [16] and specialist communication [17]; a pediatric referral study distinguished scheduling from completion [18]. None of those process measures can be reconstructed from a Table 4-63 count. A data-quality framework separates conformance and plausibility from fitness for a particular analytic use [19]. RECORD [20] and STROBE [21] supply useful reporting principles for source, selection, variables and limitations, although this is a documentary census rather than a patient-level cohort. We therefore asked: among the counts printed in the MOH 2024 twenty-cluster matrix, what fractions are on the diagonal, which unordered label pairs account for the largest off-diagonal counts, and how much of the off-diagonal total is between distinct clusters with the same Riyadh or Jeddah city name? We did not study patient access, completed care, referral efficiency, facility performance or outcomes.

Methods

Design, source and unit

We conducted a deterministic cross-sectional analysis of one official MOH 2024 yearbook workbook [1]. A frozen copy is bound to this study as official/Statistical-Yearbook-2024.xlsx, SHA-256 a482e1f88c13b24246312fdb4696e3f9c49f2dd9459d56ef585f47bb47032c83, identical to the linked MOH file at extraction. On worksheet 4-63, Table 4-63, the 400 numeric intersections occupy B6:U25, row totals V6:V25, column totals B26:U26, and the grand total V26. We retained twenty label positions, trimmed a leading space in the Hail column heading, and checked English column labels (B5:U5) against English row aliases (W6:W25) by positional order. Wording differs, for example “Riyadh Cluster I” versus “Riyadh First Health Cluster”; the row and column sequences nevertheless designate the same twenty labels. Labels were standardized for display only. Neither spelling normalization nor the official English names supplies a facility or population crosswalk.

The observation unit for all calculations was a published matrix cell, not a person, known single referral episode or hospital encounter. We refer to the source's entries as “published referral counts,” because its caption includes reverse referrals but does not define whether repeated requests or linked stages are counted once. The table does not identify accepted, denied, scheduled, completed or cancelled referrals. The twenty labels are health clusters, not Saudi Arabia's thirteen administrative regions; a city can contain more than one cluster. We made no substitution from SMARC records or another yearbook table. Source cells were read in their stored integer form; no percentages in this article are transcribed from a precomputed graphic.

Validation and orientation-invariant measures

Let M[i,j] denote the nonnegative integer at position i,j for i,j from 1 to 20 in the common label order. We asserted that each of the twenty sums over j equals its printed V total, each of the twenty sums over i equals its printed row-26 total, and their common grand sum equals V26. We separately confirmed that every cell is an integer at least zero and that each label occurs once on each axis after whitespace normalization. Passing these checks verifies transcription and internal workbook arithmetic, not the events in MOH source systems or the intended axis orientation.

The descriptive total was N = sum(M[i,j]). The diagonal count was D = sum(M[i,i]), and off-diagonal count O = N − D. We report D/N and O/N; a diagonal entry is only a same printed cluster-label position. For every two distinct labels i < j, we formed the unordered-pair count P[i,j] = M[i,j] + M[j,i]. There are 190 possible pairs, and the pair counts sum exactly to O. We sorted pairs by descending two-cell count, breaking count ties by the two printed English column labels alphabetically; Table 5 reports all 190 ranks, coordinates and source values. Both D and P are unchanged by transposing M. We do not report in-degree, out-degree, net exchange, directional arrows, path length or patient-flow centrality. The Arabic stacked headings at A4/A5 read receiving then sending clusters, while the English headings at W4/W5 read “Referred from” then “Referred to” in their stacked order. Those captions do not unambiguously assign one English directional label to numeric rows versus columns. Only an attributable MOH key can settle orientation.

The five-pair summary was chosen after seeing the matrix as an exploratory reporting cutoff, not as a source-defined or clinically meaningful threshold. We display the top-one, top-five, top-six and top-ten counts and shares to show sensitivity to that choice; in particular, we report the fifth–sixth gap. Separately, we defined four unordered pairs whose printed labels share a Riyadh or Jeddah city name: all three pairings among Riyadh I, II and III, and Jeddah I–II. Their combined count was subtracted from O to show how much of the off-diagonal total remains when these specific unlike-label pairs are set aside. This name-based partition was transparently calculated for description; no dated prospective protocol is asserted. It does not establish shared catchments, geographic boundaries, distance traveled or patients' residence. We made no inference from the remaining pairs about interregional travel. Table 1 defines the denominators; Table 2 supplies the largest source-pair positions; Table 3 presents the city-name sensitivity; Table 4 shows cutoff sensitivity. We did not calculate confidence intervals or hypothesis tests for this complete finite matrix of published cells.

Prior work and interpretation plan

Before drafting, we reviewed the full accessible 17-page 2025 SMARC article [2], checked the earlier Saudi reports [3–6] by indexed metadata and abstracts, and ran a finite PubMed/web search for the MOH matrix, cluster-pair and referral-flow terms. We did not treat abstracts as full-text verification of every contextual claim; a small claim-to-source ledger is retained for human authors. The 2024 e-consultation review [7] was used only as indirect context on access outcomes, not as validation of the matrix or its referral direction. The literature screen is not an exhaustive systematic review and cannot certify priority. We compared numerator definitions and geography rather than percentages across sources. The reporting approach adapts source-provenance and limitation items from RECORD [20] and STROBE [21], without claiming that participant selection, confounding or follow-up items apply to a table census. Exact cell locators, extraction code, the 190-pair CSV, source hash and local frozen workbook are retained in this study folder; the full 190-pair ledger is also reproduced in Table 5 of this DOCX. An external code location remains to be chosen by accountable authors.

Results

Matrix accounting and diagonal fraction

The 400 values and all forty printed marginal totals reconciled to V26 = 341,365. The twenty diagonal positions summed to 301,968, or 88.46% of N. Off-diagonal positions summed to 39,397, or 11.54%. Exactly 246 of the 380 off-diagonal *positions* were nonzero; at the unordered-pair level, 151 of 190 two-cell sums were positive. These counts describe the density of the published grid, not the number of referral

routes, hospitals or people. Table 1 shows the arithmetic and the relevant denominators. A transpose of the original matrix preserves all reported results.

Table 1. Accounting identities and orientation-invariant summary of MOH Table 4-63

Published-grid quantity	Count	Denominator and interpretation
Cluster labels on each axis	20	Same normalized label order on both axes
Matrix positions	400	20 × 20 complete cells
All published counts, N	341,365	Sum of B6:U25; equals V26
Diagonal positions, D	301,968 (88.46%)	D/N; same printed label on both axes
Off-diagonal positions, O	39,397 (11.54%)	O/N; two different printed labels
Positive off-diagonal positions	246/380	Ordered positions, direction not inferred
Positive unordered pairs	151/190	P[i,j] > 0, with P[i,j] = M[i,j] + M[j,i]

Source: official MOH 2024 yearbook worksheet 4-63, Table 4-63 [1], workbook SHA-256 in Methods. All 20 printed row totals and 20 printed column totals were recalculated exactly. Percentages use 341,365 as denominator and are rounded to two decimal places. “Diagonal” and “off-diagonal” describe label positions, not completed care or actual travel. The table includes reverse referrals; its event unit and directional axes were not independently verified.

The count's meaning is bounded by the source caption. It is defensible to say that 301,968 numbers are on like-labelled intersections. It would not be defensible to say 301,968 *patients remained within a cluster*, or that 39,397 people crossed a cluster boundary. The single N provides no rate denominator: it is not the number of patients who needed referral, the number of referral requests started, or the number of opportunities for acceptance. The diagonal result is therefore a property of the publication, not a quality-of-care indicator.

Largest unordered pairs

Summing the two opposing positions for each unlike-label pair makes the source's directional ambiguity irrelevant to the ranking. The five largest pairs contributed 17,496 of 39,397 off-diagonal counts (44.41%). The Jeddah I–II pair alone contributed 8,265 (20.98%). Riyadh I–III contributed 3,361; Riyadh II–III 2,110; Riyadh I–II 2,036; and Madinah–Tabouk 1,724. The sixth was Eastern–Al-Ahsa at 1,704, 20 fewer than the fifth; there is no tie at this display cutoff. Table 2 supplies the exact two source cells for the displayed top five, and Table 5 supplies source coordinates and both values for all 190 unordered pairs, including zero pairs. Their order within each pair follows the printed label index only and must not be read as a sending-to-receiving arrow.

Table 2. Five largest unordered pairs of different printed health-cluster labels

Unordered pair	Source cells (position counts)	Two-cell count	Share of off-diagonal O
Jeddah I–Jeddah II	G10=6,441; F11=1,824	8,265	20.98%
Riyadh I–Riyadh III	D6=340; B8=3,021	3,361	8.53%
Riyadh II–Riyadh III	D7=92; C8=2,018	2,110	5.36%
Riyadh I–Riyadh II	C6=1,871; B7=165	2,036	5.17%

Unordered pair	Source cells (position counts)	Two-cell count	Share of off-diagonal O
Madinah–Tabouk	O13=37; I19=1,687	1,724	4.38%
Five-pair sum	Two opposing positions per pair	17,496	44.41%

Source: MOH Table 4-63 [1], worksheet cells shown; denominator is all 39,397 off-diagonal published counts, not 341,365. Each pair's two positions are summed; no source-cell direction is assigned. The four Riyadh/Jeddah pairs join distinct health-cluster labels whose printed names share a city name. A sum may include reverse referrals, repeat events and unknown referral states; it is not a count of unique moving patients. The complete ranked 190-pair source-cell ledger is Table 5 of this article.

The concentration is not uniform across all unlike-label pairs: the top five occupy only five of 190 possible unordered pairs but contain 44.41% of the off-diagonal count. Because five was an exploratory display choice, Table 4 also shows how the share changes at one, six and ten pairs; there is no source-defined threshold or detected changepoint. These sums do not establish a hub-and-spoke service organization, because referral origin/destination, contributing facilities, catchment population and referral need remain unknown. In particular, the two unequal source cells in a pair must not be described as counterflows until the bilingual table heading is clarified by MOH.

Shared city-name sensitivity

Four of the top five pairs join distinct clusters whose printed labels share the Riyadh or Jeddah city name. Together they hold 15,772 counts, 40.03% of O and 4.62% of N. Removing these four name-based pairs leaves 23,625 other off-diagonal pair counts, 59.97% of O and 6.92% of N. Table 3 shows the calculation and makes the sensitivity's assumption visible. The remaining 23,625 must not be renamed “interregional transfers”: an unlike cluster label does not identify travel distance, and neither Saudi administrative-region geography nor patient residence is defined by the table.

Table 3. Distinct-cluster pairs sharing a Riyadh or Jeddah city name

Name-based pair	Two-cell count	Share of off-diagonal O
Jeddah I–Jeddah II	8,265	20.98%
Riyadh I–Riyadh III	3,361	8.53%
Riyadh II–Riyadh III	2,110	5.36%
Riyadh I–Riyadh II	2,036	5.17%
Four-pair sum	15,772	40.03%
All other off-diagonal pairs	23,625	59.97%

Source and pair formula as Table 2 [1]. “Shared city name” is a partition based on the printed label strings, not a verified catchment, municipal or distance classification. Counts for the four pairs are subtracted once each from O = 39,397. The residual equals 6.92% of N = 341,365. Percentages were calculated from unrounded counts; displayed component percentages may not sum exactly. Neither subset is an accepted/completed referral or unique-patient denominator.

Descriptive rank-cutoff sensitivity

The five-pair cutoff captures nearly half the off-diagonal published counts, but its exact percentage changes materially if one more pair is included. One pair accounts for 20.98%, five for 44.41%, six for 48.73% and ten for 62.03%. The fifth and sixth sums differ by 20 counts, which is only 0.05 percentage points of O. The rank ordering is exact for the released workbook under the two-cell rule; neither the fifth/sixth boundary nor

a ten-pair boundary has intrinsic clinical or policy meaning. Table 4 reports the values without treating the proportions as confidence intervals or temporal trends.

Table 4. Sensitivity of off-diagonal share to exploratory top-k display cutoff

Highest-ranked unordered pairs, k	Cumulative two-cell count	Share of O = 39,397
1	8,265	20.98%
5	17,496	44.41%
6	19,200	48.73%
10	24,439	62.03%

Source: all 190 two-cell sums from MOH Table 4-63 [1], ranked by descending sum with printed-label alphabetical tie-break; Table 5 provides every pair and both source-cell coordinates. Shares use the full off-diagonal O = 39,397; values are rounded to two decimals. Fifth-ranked Madinah–Tabouk = 1,724 and sixth-ranked Eastern–Al-Ahsa = 1,704, a difference of 20. The displayed cutoffs are exploratory descriptions, not prespecified source thresholds or separate statistical groups. Source: MOH 2024 yearbook Table 4-63 [1], workbook SHA-256 a482e1f88c13b24246312fdb4696e3f9c49f2dd9459d56ef585f47bb47032c83. Each row sums source cells M[i,j]+M[j,i] for i < j; totals rank descending, with printed English labels breaking ties. The 190 pair sums equal O = 39,397, with 39 zero pairs. Cell order has no verified direction or unique-patient meaning.

Table 5. Complete 190-pair source-cell ledger

All 190 unordered pairs, including 39 zero pairs, are reproduced here from official worksheet 4-63 [1]; this in-manuscript ledger is not a public machine-readable deposit.

Table A1. All 190 unordered cluster-label pairs ranked by their two printed source positions

Rank	Printed cluster-label pair	Cell 1	Cell 2	Pair sum
1	Jeddah Cluster I / Jeddah Cluster II	G10 6,441	F11 1,824	8,265
2	Riyadh Cluster I / Riyadh Cluster III	D6 340	B8 3,021	3,361
3	Riyadh Cluster II / Riyadh Cluster III	D7 92	C8 2,018	2,110
4	Riyadh Cluster I / Riyadh Cluster II	C6 1,871	B7 165	2,036
5	Madinah / Tabouk	O13 37	I19 1,687	1,724
6	Eastern / Al-Ahsa	L15 485	K16 1,219	1,704
7	Eastern / Hafr Al-Baten	M15 102	K17 1,431	1,533
8	Riyadh Cluster II / Al Jouf	U7 11	C25 1,290	1,301
9	The Holy Capital / Jeddah Cluster II	G9 747	E11 482	1,229
10	Riyadh Cluster II / Asir	N7 13	C18 1,163	1,176

Rank	Printed cluster-label pair	Cell 1	Cell 2	Pair sum
11	The Holy Capital / Ta`if	H9 133	E12 912	1,045
12	The Holy Capital / Al-Bahah	T9 417	E24 612	1,029
13	The Holy Capital / Jeddah Cluster I	F9 338	E10 639	977
14	Riyadh Cluster II / Northern Borders	Q7 8	C21 867	875
15	Riyadh Cluster II / Qassim	J7 105	C14 631	736
16	Riyadh Cluster I / Al Jouf	U6 41	B25 533	574
17	Riyadh Cluster II / Najran	S7 10	C23 563	573
18	Riyadh Cluster II / Hafr Al-Baten	M7 6	C17 561	567
19	Riyadh Cluster II / Tabouk	O7 12	C19 502	514
20	Riyadh Cluster II / Hail	P7 12	C20 491	503
21	Riyadh Cluster II / Jazan	R7 4	C22 475	479
22	Northern Borders / Al Jouf	U21 209	Q25 156	365
23	Riyadh Cluster I / Northern Borders	Q6 29	B21 256	285
24	Qassim / Al-Ahsa	L14 0	J16 258	258
25	Riyadh Cluster I / Asir	N6 61	B18 196	257
26	Ta`if / Asir	N12 213	H18 11	224
27	The Holy Capital / Asir	N9 29	E18 195	224
28	Eastern / Northern Borders	Q15 2	K21 215	217
29	The Holy Capital / Al Jouf	U9 0	E25 214	214
30	Riyadh Cluster II / Madinah	I7 4	C13 208	212
31	Riyadh Cluster I / Qassim	J6 31	B14 157	188
32	Asir / Najran	S18 33	N23 148	181

Rank	Printed cluster-label pair	Cell 1	Cell 2	Pair sum
33	The Holy Capital / Eastern	K9 173	E15 6	179
34	The Holy Capital / Jazan	R9 10	E22 166	176
35	Riyadh Cluster I / Tabouk	O6 12	B19 160	172
36	Riyadh Cluster II / Eastern	K7 5	C15 167	172
37	The Holy Capital / Madinah	I9 10	E13 154	164
38	Qassim / Hail	P14 11	J20 148	159
39	The Holy Capital / Tabouk	O9 4	E19 153	157
40	Riyadh Cluster II / Al-Bahah	T7 2	C24 154	156
41	Riyadh Cluster I / Najran	S6 44	B23 111	155
42	Jeddah Cluster I / Ta`if	H10 111	F12 39	150
43	Riyadh Cluster I / Hafr Al-Baten	M6 19	B17 130	149
44	Riyadh Cluster II / Al-Ahsa	L7 5	C16 121	126
45	Eastern / Al Jouf	U15 3	K25 116	119
46	The Holy Capital / Qassim	J9 113	E14 2	115
47	Eastern / Tabouk	O15 1	K19 107	108
48	The Holy Capital / Northern Borders	Q9 0	E21 103	103
49	Eastern / Asir	N15 8	K18 92	100
50	Madinah / Al Jouf	U13 1	I25 98	99
51	Jeddah Cluster II / Ta`if	H11 5	G12 89	94
52	Riyadh Cluster I / Hail	P6 11	B20 79	90
53	Asir / Jazan	R18 28	N22 54	82
54	Hail / Al Jouf	U20 14	P25 67	81
55	Hafr Al-Baten / Northern Borders	Q17 1	M21 78	79

Rank	Printed cluster-label pair	Cell 1	Cell 2	Pair sum
56	Qassim / Jazan	R14 0	J22 79	79
57	Ta`if / Al-Bahah	T12 2	H24 77	79
58	Eastern / Jazan	R15 3	K22 65	68
59	Riyadh Cluster II / The Holy Capital	E7 3	C9 65	68
60	Madinah / Qassim	J13 63	I14 3	66
61	Riyadh Cluster I / Jazan	R6 9	B22 48	57
62	Ta`if / Qassim	J12 57	H14 0	57
63	Asir / Al-Bahah	T18 25	N24 31	56
64	Al-Ahsa / Hafr Al-Baten	M16 2	L17 48	50
65	Jeddah Cluster II / Madinah	I11 9	G13 40	49
66	Riyadh Cluster II / Ta`if	H7 2	C12 47	49
67	Qassim / Asir	N14 0	J18 47	47
68	Hail / Northern Borders	Q20 3	P21 43	46
69	The Holy Capital / Najran	S9 7	E23 38	45
70	Madinah / Hail	P13 1	I20 42	43
71	Riyadh Cluster II / Jeddah Cluster I	F7 3	C10 31	34
72	Riyadh Cluster I / Eastern	K6 12	B15 18	30
73	Riyadh Cluster I / Madinah	I6 2	B13 28	30
74	Qassim / Eastern	K14 23	J15 5	28
75	Riyadh Cluster I / Al-Ahsa	L6 9	B16 17	26
76	Riyadh Cluster II / Jeddah Cluster II	G7 0	C11 26	26
77	Riyadh Cluster I / Al-Bahah	T6 4	B24 21	25
78	Qassim / Al Jouf	U14 3	J25 20	23
79	Jeddah Cluster I / Al-Bahah	T10 5	F24 14	19

Rank	Printed cluster-label pair	Cell 1	Cell 2	Pair sum
80	Eastern / Hail	P15 0	K20 17	17
81	Qassim / Northern Borders	Q14 1	J21 15	16
82	Riyadh Cluster III / Asir	N8 7	D18 8	15
83	Tabouk / Al Jouf	U19 1	O25 14	15
84	Qassim / Najran	S14 0	J23 14	14
85	Riyadh Cluster III / Eastern	K8 14	D15 0	14
86	Riyadh Cluster III / Qassim	J8 14	D14 0	14
87	Jeddah Cluster II / Al-Bahah	T11 5	G24 8	13
88	Madinah / Eastern	K13 11	I15 2	13
89	Riyadh Cluster III / Al Jouf	U8 0	D25 13	13
90	Jeddah Cluster II / Eastern	K11 11	G15 1	12
91	Qassim / Al-Bahah	T14 0	J24 12	12
92	Riyadh Cluster I / Ta`if	H6 3	B12 9	12
93	Madinah / Jazan	R13 0	I22 11	11
94	Riyadh Cluster I / The Holy Capital	E6 4	B9 7	11
95	Madinah / Najran	S13 3	I23 7	10
96	Qassim / Hafr Al-Baten	M14 2	J17 7	9
97	Ta`if / Madinah	I12 7	H13 1	8
98	Eastern / Najran	S15 1	K23 6	7
99	Riyadh Cluster I / Jeddah Cluster II	G6 1	B11 6	7
100	Ta`if / Eastern	K12 6	H15 1	7
101	Eastern / Al-Bahah	T15 0	K24 6	6
102	Jazan / Najran	S22 5	R23 1	6
103	Jeddah Cluster I / Asir	N10 5	F18 1	6
104	Jeddah Cluster II / Jazan	R11 4	G22 2	6

Rank	Printed cluster-label pair	Cell 1	Cell 2	Pair sum
105	Madinah / Asir	N13 0	I18 6	6
106	Al-Ahsa / Jazan	R16 2	L22 3	5
107	Jeddah Cluster I / Madinah	I10 4	F13 1	5
108	Al-Ahsa / Asir	N16 0	L18 4	4
109	Riyadh Cluster I / Jeddah Cluster I	F6 2	B10 2	4
110	Ta`if / Najran	S12 0	H23 4	4
111	Al-Ahsa / Hail	P16 0	L20 3	3
112	Al-Ahsa / Tabouk	O16 0	L19 3	3
113	Hafr Al-Baten / Al Jouf	U17 3	M25 0	3
114	Jeddah Cluster I / Eastern	K10 3	F15 0	3
115	Jeddah Cluster I / Najran	S10 1	F23 2	3
116	Jeddah Cluster II / Najran	S11 3	G23 0	3
117	Jeddah Cluster II / Northern Borders	Q11 0	G21 3	3
118	Madinah / Northern Borders	Q13 0	I21 3	3
119	Tabouk / Hail	P19 3	O20 0	3
120	Tabouk / Jazan	R19 3	O22 0	3
121	Tabouk / Northern Borders	Q19 0	O21 3	3
122	The Holy Capital / Al-Ahsa	L9 0	E16 3	3
123	Al-Ahsa / Al Jouf	U16 0	L25 2	2
124	Al-Ahsa / Najran	S16 0	L23 2	2
125	Hafr Al-Baten / Hail	P17 2	M20 0	2
126	Jeddah Cluster I / Al Jouf	U10 0	F25 2	2
127	Jeddah Cluster II / Asir	N11 2	G18 0	2
128	Jeddah Cluster II / Tabouk	O11 0	G19 2	2

Rank	Printed cluster-label pair	Cell 1	Cell 2	Pair sum
129	Qassim / Tabouk	O14 1	J19 1	2
130	Riyadh Cluster III / Al-Ahsa	L8 1	D16 1	2
131	Riyadh Cluster III / Hafr Al-Baten	M8 0	D17 2	2
132	Hafr Al-Baten / Al-Bahah	T17 1	M24 0	1
133	Hail / Jazan	R20 0	P22 1	1
134	Jazan / Al-Bahah	T22 1	R24 0	1
135	Jeddah Cluster I / Al-Ahsa	L10 0	F16 1	1
136	Jeddah Cluster II / Al Jouf	U11 1	G25 0	1
137	Jeddah Cluster II / Hafr Al-Baten	M11 1	G17 0	1
138	Jeddah Cluster II / Hail	P11 0	G20 1	1
139	Jeddah Cluster II / Qassim	J11 1	G14 0	1
140	Madinah / Al-Bahah	T13 0	I24 1	1
141	Madinah / Hafr Al-Baten	M13 1	I17 0	1
142	Najran / Al-Bahah	T23 1	S24 0	1
143	Riyadh Cluster III / Al-Bahah	T8 0	D24 1	1
144	Riyadh Cluster III / Jeddah Cluster II	G8 0	D11 1	1
145	Riyadh Cluster III / Madinah	I8 0	D13 1	1
146	Riyadh Cluster III / Northern Borders	Q8 0	D21 1	1
147	Riyadh Cluster III / Ta`if	H8 1	D12 0	1
148	Riyadh Cluster III / The Holy Capital	E8 0	D9 1	1
149	Tabouk / Najran	S19 1	O23 0	1
150	The Holy Capital / Hafr Al-Baten	M9 1	E17 0	1
151	The Holy Capital / Hail	P9 0	E20 1	1

Rank	Printed cluster-label pair	Cell 1	Cell 2	Pair sum
152	Al-Ahsa / Al-Bahah	T16 0	L24 0	0
153	Al-Ahsa / Northern Borders	Q16 0	L21 0	0
154	Al-Bahah / Al Jouf	U24 0	T25 0	0
155	Asir / Al Jouf	U18 0	N25 0	0
156	Asir / Hail	P18 0	N20 0	0
157	Asir / Northern Borders	Q18 0	N21 0	0
158	Asir / Tabouk	O18 0	N19 0	0
159	Hafr Al-Baten / Asir	N17 0	M18 0	0
160	Hafr Al-Baten / Jazan	R17 0	M22 0	0
161	Hafr Al-Baten / Najran	S17 0	M23 0	0
162	Hafr Al-Baten / Tabouk	O17 0	M19 0	0
163	Hail / Al-Bahah	T20 0	P24 0	0
164	Hail / Najran	S20 0	P23 0	0
165	Jazan / Al Jouf	U22 0	R25 0	0
166	Jeddah Cluster I / Hafr Al-Baten	M10 0	F17 0	0
167	Jeddah Cluster I / Hail	P10 0	F20 0	0
168	Jeddah Cluster I / Jazan	R10 0	F22 0	0
169	Jeddah Cluster I / Northern Borders	Q10 0	F21 0	0
170	Jeddah Cluster I / Qassim	J10 0	F14 0	0
171	Jeddah Cluster I / Tabouk	O10 0	F19 0	0
172	Jeddah Cluster II / Al-Ahsa	L11 0	G16 0	0
173	Madinah / Al-Ahsa	L13 0	I16 0	0
174	Najran / Al Jouf	U23 0	S25 0	0
175	Northern Borders / Al-Bahah	T21 0	Q24 0	0
176	Northern Borders / Jazan	R21 0	Q22 0	0

Rank	Printed cluster-label pair	Cell 1	Cell 2	Pair sum
177	Northern Borders / Najran	S21 0	Q23 0	0
178	Riyadh Cluster III / Hail	P8 0	D20 0	0
179	Riyadh Cluster III / Jazan	R8 0	D22 0	0
180	Riyadh Cluster III / Jeddah Cluster I	F8 0	D10 0	0
181	Riyadh Cluster III / Najran	S8 0	D23 0	0
182	Riyadh Cluster III / Tabouk	O8 0	D19 0	0
183	Ta`if / Al Jouf	U12 0	H25 0	0
184	Ta`if / Al-Ahsa	L12 0	H16 0	0
185	Ta`if / Hafr Al-Baten	M12 0	H17 0	0
186	Ta`if / Hail	P12 0	H20 0	0
187	Ta`if / Jazan	R12 0	H22 0	0
188	Ta`if / Northern Borders	Q12 0	H21 0	0
189	Ta`if / Tabouk	O12 0	H19 0	0
190	Tabouk / Al-Bahah	T19 0	O24 0	0

Discussion

Principal finding and what it does not show

The 2024 MOH referral table has a pronounced diagonal and an off-diagonal distribution in which a few unordered cluster pairs carry many published counts. The public diagonal observation alone adds little to the published table; external e-consultation literature [7]does not establish the meaning of these cells; the audit's incremental contribution is the complete, source-addressed 190-pair decomposition plus a transparent city-name and rank-cutoff sensitivity. A reader can recompute 17,496 as the five largest two-cell sums, check their rank against every other pair in Table 5, and reproduce 15,772 as the four Riyadh/Jeddah name-sharing pairs. These are descriptions of a 400-cell aggregate publication. They are not estimates of a hidden patient population and do not justify uncertainty intervals as though the printed matrix were a random sample.

The large Jeddah I–II and Riyadh I–III two-cell totals should prompt questions, not causal explanations. One plausible hypothesis is proximity or shared tertiary services; another is the way source systems assign clusters to facilities; still others include reverse-referral accounting, repeated requests, phased referrals or different center types. The table supplies no facility-level map, no timestamped referral sequence and no disposition. A diagonal entry could represent a transfer between two facilities under one cluster label. Conversely, an off-diagonal entry between two Riyadh-named clusters does not prove that the referring

and receiving facilities are near each other. The results do not distinguish clinical need, care coordination, local gatekeeping, administrative assignment or data-entry practice. The source owner is the appropriate party to define each event and direction before such interpretations are attempted.

The closest Saudi study, Alharbi and colleagues' 2025 SMARC analysis, tabulated 372,926 e-referrals in 2023 and 382,219 in 2024, 755,145 altogether, using thirteen administrative regions and multiple workflow attributes [2]. Its overall 87.52% “internal” designation concerns the same *administrative region* across both years. Our 88.46% diagonal is the same *health-cluster label* in one MOH 2024 government-hospital matrix that explicitly includes reverse referrals. Although the two percentages look alike, neither numerator, denominator nor geographic criterion is shown to match. Even 382,219 versus 341,365 in 2024 cannot be interpreted as missing or extra referrals without a documented mapping between platforms, sectors, reverse-referral inclusion and extraction dates. The 2020–2021 SMARC internal/external and acceptance analyses [3–5] similarly ask different questions. Their richer records support questions this public table cannot answer; this paper supplies a transparent fine-grained pair profile that those articles do not display.

Published referral-network studies clarify a further trap. A shared-patient link can be inferred from longitudinal claims or encounters, but that link is not necessarily a formal referral [8,9,13,14]. A referral path can order encounters for the *same person* [10]; this matrix cannot. Postacute referral concentration can be related to spending only because linked administrative records include payment and beneficiaries [11,12]; here there is no cost or outcome field. Thus even familiar network terms such as “hub,” “centrality,” or “flow” would imply more than the present data. We used “unordered pair” and “published count” to preserve what is actually identifiable. If the axis orientation is later documented, a directed analysis could be conducted as a new study, ideally with event-level validation and a clear policy question.

Data quality and clinical claim ceiling

The workbook passes internal arithmetic checks: every cell is nonnegative, both marginal directions reconcile, and the national total agrees. This is useful for reproducibility but weak evidence of clinical validity. The framework of Kahn and colleagues separates data conformance from fitness for a particular use [19]. The present matrix is fit for the narrow purpose of describing printed count placement, not for scoring whether referrals worked. Electronic referral implementations show that completeness of clinical information, coordination and communication can be measured [15–17]; scheduling and completion are demonstrably different endpoints [18]. None can be derived from B6:U25. Even the word “referral” here should remain tethered to the yearbook caption, rather than expanded into completed specialty care.

The bilingual header deserves a dedicated source-owner query. Arabic labels on the left and English labels on the right of the stacked heading place “referred from” and “referred to” in an order that does not unambiguously tell an outside analyst which numeric axis is origin. The bilingual row aliases and English column headings align by ordered cluster identity, despite different wording; that supports the positional diagonal but cannot settle which off-diagonal position is i-to-j. Unordered two-cell sums survive an axis reversal exactly; marginal totals can be verified without calling one a source or destination. A nonexpert may find this caution overly conservative, but a wrong orientation would reverse every directional interpretation while leaving all arithmetic checks intact. No increase in model complexity can repair a missing axis definition.

The city-name sensitivity is similarly descriptive. We identified the four pairs by a stated label rule after inspecting the grid; no prospective selection is claimed. It is reasonable to note that the two distinct labels

in each pair share “Riyadh” or “Jeddah,” and unreasonable to infer that the other 23,625 counts crossed administrative-region boundaries. Cluster polygons, referring-facility addresses and patient residence would be required for distance or catchment analysis. Moreover, the health-cluster labels are organizational units; a request counted at one site could concern care delivered at another. The association of label concentration with purchasing design in the online commentary [7] remains an untested proposition for a linked, defined dataset, not an inference this grid can test.

Strengths, limitations and next evidence

This audit reads the official workbook rather than a secondary graph; fixes the exact 400-cell universe and SHA-256; validates 40 printed marginals; discloses the pair formula and two-cell locators for all 190 pairs; and uses a statistic invariant to transposition. The complete source table is publicly locatable and the calculations require only sums and divisions. We did not fit a graph model, calculate uncertainty on a census, or claim an “optimal” concentration threshold. The five-pair cutoff was exploratory and the top-one, six and ten comparisons expose its sensitivity. The result remains reproducible if all rows and columns are transposed, and its count-based denominators are explicit in Tables 1–4 and Table 5.

The chief limitation is conceptual: the publication supplies a number for a cluster pair without a record-level data dictionary. We do not know its event unit, repeat handling, inclusion of reverse referrals, source and destination facility types, whether entries are attempted, accepted or completed, or when they were entered. We have no linked patient identities, procedure types, referral reasons, triage status, dates, capacity, distance or outcomes. A single 2024 snapshot cannot establish a temporal pattern or effect of Saudi health-sector reform. Source row/column labels do not certify catchment boundaries. The English spelling of a cluster is not an administrative-region identifier. We have not obtained MOH confirmation of the directional axes; an erroneous axis inference would be particularly consequential for policy. The finite literature screen and the 2025 SMARC full-text comparison do not certify first publication of a cluster analysis, especially in Arabic or grey literature. Table 5 makes the exact pair ordering inspectable in this DOCX, but code and CSV remain internal rather than a persistent public supplement.

An attributable MOH or referral-data-owner response should specify: the event counted in one cell; whether reverse referrals share the same inclusion rule; whether repeat requests, cancellations and multi-step referrals are counted more than once; whether source and destination are assigned by organization, facility or patient location; what acceptance or completion stage is included; the exact row/column orientation; and whether each cluster label has a dated organizational map. If those definitions support a linked event-level analysis, a later study can investigate direction, network paths and patient outcomes. If they remain unavailable, the present manuscript can still accurately report a published-table topology provided a human editor accepts that limited contribution.

Conclusion

The 2024 MOH twenty-cluster referral matrix prints 341,365 counts, of which 301,968 lie on its diagonal and 39,397 off it. Five unordered cluster pairs account for 17,496 off-diagonal counts under an exploratory cutoff; four pairs between distinct clusters whose printed labels share Riyadh or Jeddah account for 15,772. These source-cell facts are invariant to transposing the matrix. They neither establish a route taken by a patient nor measure acceptance, completed care, access, quality or health outcomes. A directional or performance interpretation requires the source owner's definition of the counted event and axes.

Declarations

Ethics and consent: Not applicable; this work uses publicly available literature or aggregate published information and does not involve newly collected individual patient data.

Data availability: All underlying cited sources are available online, as identified in the references.

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Conflicts of interest: The authors declare no competing interests.

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